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The Traveling Salesman Problem مسألة البائع المتجول

وهي مسألة ايجاد اقصر طريق مغلق بين مجموعة من المدن

The Traveling Salesman Problem (TSP) is one of the well-known problems for finding an optimal path. The TSP problem deals with finding the shortest (closed) tour in an *n*-city situation, where each city is visited exactly once before returning back to the starting point.

Mathematical problems related to the Traveling salesman problems were treated in the 1800's by the Irish Mathematician W. R. Hamilton and by the British Mathematician Thomas Kirkman. Hamilton's Icosian Game was a recreational puzzle based on finding a Hamiltonian cycle. The general form of the TSP appears to have been first studied by mathematicians during the 1930's in Vienna and at Harvard, notably by Karl Menger, In the 1950's and 1960's, the problem became increasingly popular in scientific circles in Europe and the U.S. Notable contributions.

The general formulation of TSP model is defined by two pieces of data:

1- The number of cities, *n*.
2- The distances d_{ij} between cities *i* and *j* ($d_{ij} = \infty$ if cities *i* and j are not linked.

	1	2	3	.....	n
1	———	d_{12}	d_{13}	.....	d_{1n}
2	d_{21}	———	d_{23}	.....	d_{2n}
3	d_{31}	d_{32}	———	.....	d_{3n}
.	.	110	.	———	.
.	.		.		.
.	.		.		.
n	d_{n1}	d_{n2}	d_{n3}	.....	———

A TSP is **symmetric** if $d_{ij} = d_{ji}$ for all i and j. Otherwise, the TSP is called **asymmetric**.

Define $x_{ij} = \begin{cases} 1 \text{ , } if\ city\ j\ is\ reached\ from\ city\ i \\ 0 \text{ , } otherwise \end{cases}$

The TSP model is given as

$$Min \quad z = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}, \quad d_{ij} = \infty\ for\ all\ i = j$$

$$Subject\ to \quad \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \dots, n \ , i \neq j$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \dots, n \ , i \neq j$$

$$x_{ij} = 0 \ or \ 1$$

TSP solution

i- A straightforward way to solve TSP is **Exhaustive enumeration**. **The maximum number of tours in an *n*-city is (*n*-1)!**

We are going to find all the tours by starting from certain city. The steps are

1- Choose first city α, then choose the smallest distance in the row, say city β.
2- Go to the row of city β and choose the smallest distance, but without repeating the previous city.
3- Continue the process to all cities till you rerun to city α.
4- The smallest routs are the optimal solution.

Example1: A book salesman who lives in Baghdad must call once a month on four customers located in Kufa, Basra, Mosel, and Karbala. The following table gives the distances in miles among different cities. The objective is to minimize the total distance travelled by the salesman. Find all the tours that give an optimal solution.

Miles between cities					
	Baghdad 1	**Kufa 2**	**Basra 3**	**Mosel 4**	**Karbala 5**
Baghdad 1	∞	**120**	**220**	**150**	**210**
Kufa 2	**120**	∞	**80**	**110**	**130**
Basra 3	**220**	**80**	∞	**160**	**185**
Mosel 4	**150**	**110**	**160**	∞	**190**
Karbala 5	**210**	**130**	**185**	**190**	∞

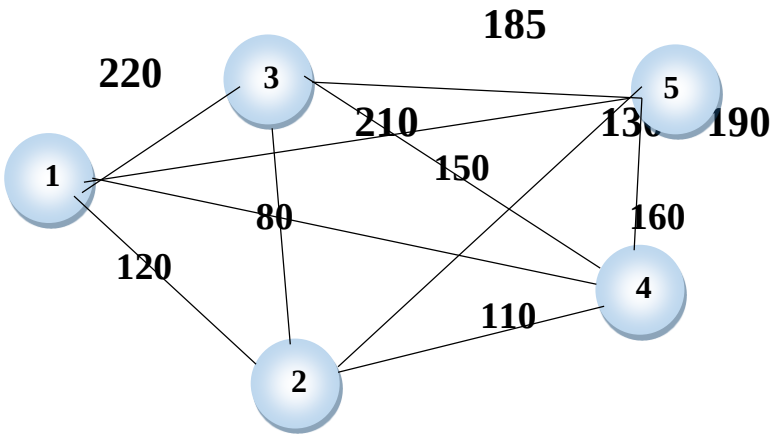
Solution: We start finding all the possible routs and choose the minimum one.

Start city	**Tour**	**Length**
1	1→2 →3 →4 →5 →1	120+80+160+190+210 = 760
2	**2→3 →4 →1 →5 →2**	**80+160+150+210+130 = 730**
3	3→2 →4 →1 →5 →3	80+110+150+210+220 = 770
4	4→2 →3 →5 →1 →4	110+80+185+210+150 = 735
5	**5→2 →3 →4 →1 →5**	**130+80+160+150+210 = 730**

Also the reverse of case 2 is

2	**2→5 →1 →4 →3 →2**	**130+210+150+160+80=730**

$$Min \quad z = \sum_{i=1}^{5}\sum_{j=1}^{5} d_{ij}x_{ij} = 730$$



Example2: Circuit boards are fitted with holes for mounting different electric components. The holes are drilled with a movable drill. The following table provides the distances (in centimeters) between pairs of 6 holes of a specific circuit board.

centimeters between holes d_{ij}					
∞	**1.2**	**0.5**	**2.6**	**4.1**	**3.2**
1.2	∞	**3.4**	**4.6**	**2.9**	**5.2**
0.5	**3.4**	∞	**3.5**	**4.6**	**6.2**
2.6	**4.6**	**3.5**	∞	**3.8**	**0.9**
4.1	**2.9**	**4.6**	**3.8**	∞	**1.9**
3.2	**5.2**	**6.2**	**0.9**	**1.9**	∞

Solution: We start finding all the possible routs and choose the minimum one.

Start city	**Tour**	**Length**
1	1→3 →2 →5 →6 →4→1	0.5+3.4+2.9+1.9+0.9+2.6= 12.2
2	**2→1 →3 →4 →6 →5→2**	**1.2+0.5+3.5+0.9+1.9+2.9=10.9**
3	**3→1 →2 →5 →6 →4→3**	**0.5+1.2+2.9+1.9+0.9+3.5=10.9**
4	**4→6 →5 →2 →1 →3→4**	**0.9+1.9+2.9+1.2+0.5+3.5=10.9**
5	5→6 →4 →1 →3 →2→5	1.9+0.9+2.6+0.5+3.4+2.9=12.2
6	6→4 →1 →3 →2 →5→6	0.9+2.6+0.5+3.4+2.9+1.9=12.2

$$Min \quad z = \sum_{i=1}^{6}\sum_{j=1}^{6} d_{ij}x_{ij} = 10.9$$

H.W: Find the optimum tour for the traveling salesman problem using exhaustive enumeration method if the cost matrix is given by table:

	1	2	3	4	5
1	….	35	80	105	165
2	35	…..	45	20	80
3	80	45	…..	30	75
4	105	20	30	….	60
5	165	80	75	60	….

ii- **Develop a "near-optimal" algorithm for the traveling salesman problem**

Develop the nearest-neighbor method, based on the principle of sequentially selecting the cheapest remaining link such that its inclusion does not complete a circuit too soon.

Step1: Locate the smallest element in the cost matrix, circle it and include the corresponding link in the tour.

Step2: If the newly circled element is $\boldsymbol{d_{ij}}$ replace all other elements in the ith row and all other elements in the jth column, as well as the transposed element $\boldsymbol{d_{ij}}$ by a prohibitively large number.

Step3: Locate the smallest encircled element in the latest cost matrix. Attentively adjoin its corresponding link to the (incomplete) tour. If the resulting tour is feasible, circle the designated cost and go to step 5.

Step4: If the resulting tour is infeasible, remove the latest link from the tour and replace its corresponding cost by a prohibitively large number. Go to step 3.

Step5: Determine whether the tour is complete. If so, accept it as the near-optimal one. If not, go to step 2.

Example: Use the nearest–neighbor method to find a near-optimal traveling salesman problem tour for the following table:

Miles between cities					
	Baghdad 1	**Kufa 2**	**Basra 3**	**Mosel 4**	**Karbala 5**
Baghdad 1	... ...	**120**	**220**	**150**	**210**
Kufa 2	**120**	... ...	**80**	**110**	**130**
Basra 3	**220**	**80**	... ...	**160**	**185**
Mosel 4	**150**	**110**	**160**	... ...	**190**
Karbala 5	**210**	**130**	**185**	**190**	... ...

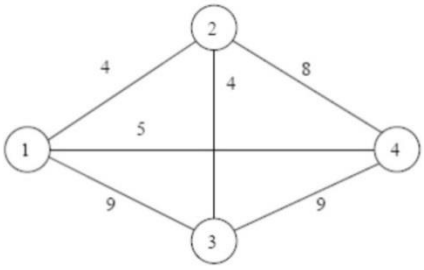
Step1: We first replace the dotted entries in the cost matrix with a prohibitively large number (1000)

Miles between cities					
	Baghdad 1	**Kufa 2**	**Basra 3**	**Mosel 4**	**Karbala 5**
Baghdad 1	**1000**	**120**	**1000**	**150**	**210**
Kufa 2	**1000**	**1000**	**80**	**1000**	**1000**
Basra 3	**220**	**80**	**1000**	**160**	**185**
Mosel 4	**150**	**110**	**1000**	**1000**	**190**
Karbala 5	**210**	**130**	**1000**	**190**	**1000**

Miles between cities					
	Baghdad 1	**Kufa 2**	**Basra 3**	**Mosel 4**	**Karbala 5**
Baghdad 1	**1000**	**1000**	**1000**	**1000**	**210**
Kufa 2	**1000**	**1000**	**80**	**1000**	**1000**
Basra 3	**1000**	**1000**	**1000**	**160**	**1000**
Mosel 4	**150**	**1000**	**1000**	**1000**	**1000**
Karbala 5	**1000**	**130**	**1000**	**1000**	**1000**

Optimal Solution: 2→3 →4 →1 →5 →2 80+160+150+210+130=730

H.W: Use the nearest–neighbor method to find a near-optimal traveling salesman problem tour, if the cost is given by figure:



H.W: Use the nearest–neighbor method to find a near-optimal traveling salesman problem tour, if the cost matrix is given by table:

	1	2	3	4	5
1	….	35	80	105	165
2	35	…..	45	20	80
3	80	45	…..	30	75
4	105	20	30	….	60
5	165	80	75	60	….

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